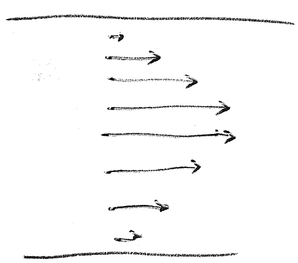
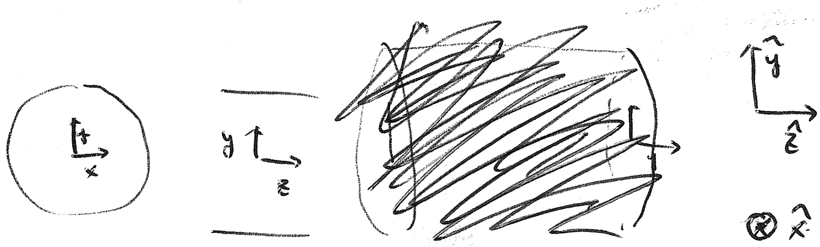
GAUSS'S LAW

CONSIDER A PIPE WITH WATER FLOWING INSIDE OF IT

CROSS SECTION



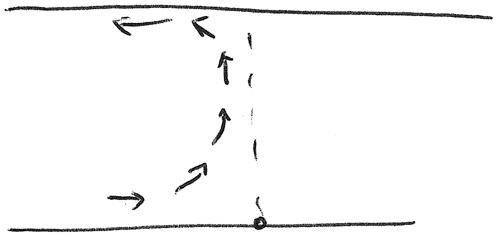
VELOCITY DISTRIBUTION



$V_z(x,y)$: VELOCITY OF FLUID IN Z DIRECTION

~~AVERAGE VELOCITY~~

FLOW = $\int V_z(x,y)\,dx\,dy$

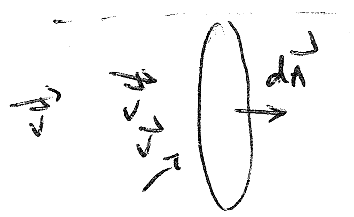


FLOW HERE IS 0

SINCE $\int V_z \, dx \, dy = 0$

IN VECTOR FORM

$$V_z = \vec{v} \cdot d\vec{A}$$

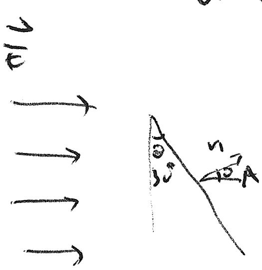


FLOW = "FLUX" $= \int_{SURFACE} \vec{v} \cdot d\vec{A}$

$$\text{ELECTRIC FLUX} = \int_{SURFACE} \vec{E} \cdot d\vec{A} = \oint \vec{E} \cdot \hat{n} \cdot ds = \Phi$$

EXAMPLE #1

UNIFORM ELECTRIC FIELD TILTED PLANE

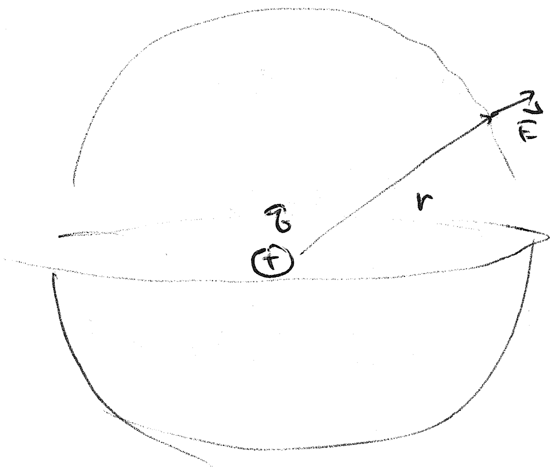


$$\vec{E} \cdot \vec{n} = |\vec{E}||\hat{n}| \cos\theta$$

$$= \frac{\sqrt{3}}{2} E$$

$$\Phi = \oint \frac{\sqrt{3}}{2} E \, d\vec{s} \neq \boxed{\frac{\sqrt{3}}{2} EA}$$

EXAMPLE # 1



FLUX THRU SPHERE

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,\hat{r}$$

$$\text{FLUX} - \Phi = \int \vec{E}\cdot d\vec{A}$$

$$\left[\vec{E}\cdot d\vec{A} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,d\vec{A}\right]$$

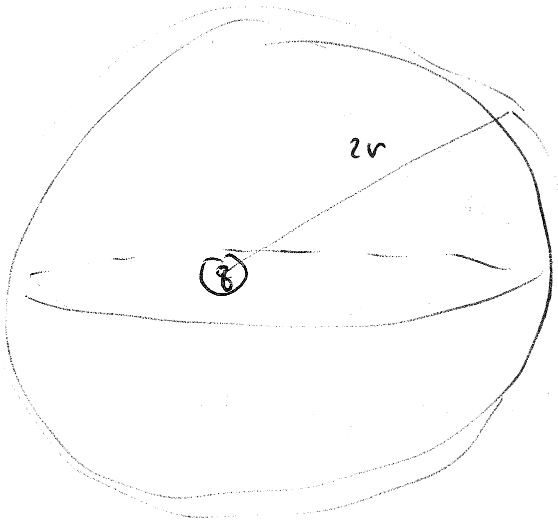
$$\Phi = \int \vec{E}\cdot d\vec{A} = |E| \int d\vec{A}$$

$$= |\vec{E}|\cdot\vec{A}$$

$$= |\vec{E}|\cdot 4\pi r^2$$

$$\Phi = \frac{q}{\varepsilon_0}$$

EXAMPLE #3



$$\Phi = |E| A \Rightarrow$$

$$|E| = \frac{1}{4\pi\varepsilon_0} \frac{q}{(2r)^2}$$

$$A = (2r)^2 4\pi \Rightarrow$$

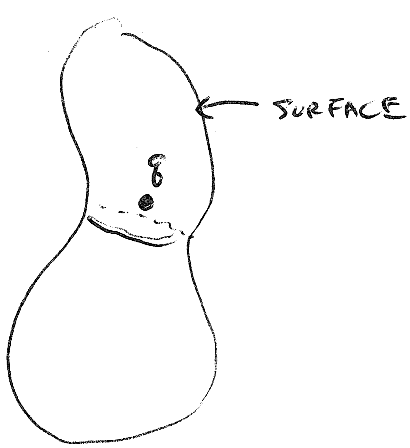
$$|E| A = \frac{1}{\cancel{4\pi}\varepsilon_0} \frac{q}{\cancel{(2r)^2}} \cancel{(2r)^2}\, \cancel{4\pi}$$

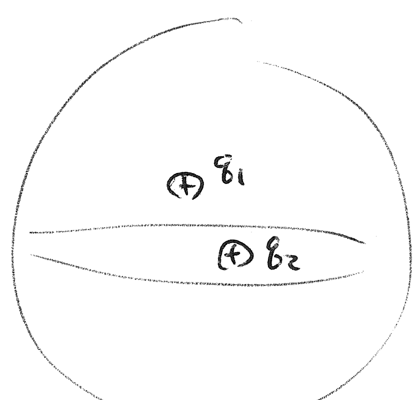
$$= \frac{q}{\varepsilon_0}$$

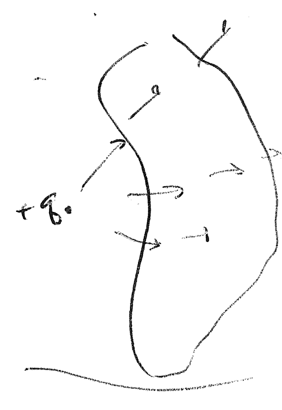
FRIDAY

GAUSS'S LAW

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{ENCLOSED}}{\varepsilon_0}$$

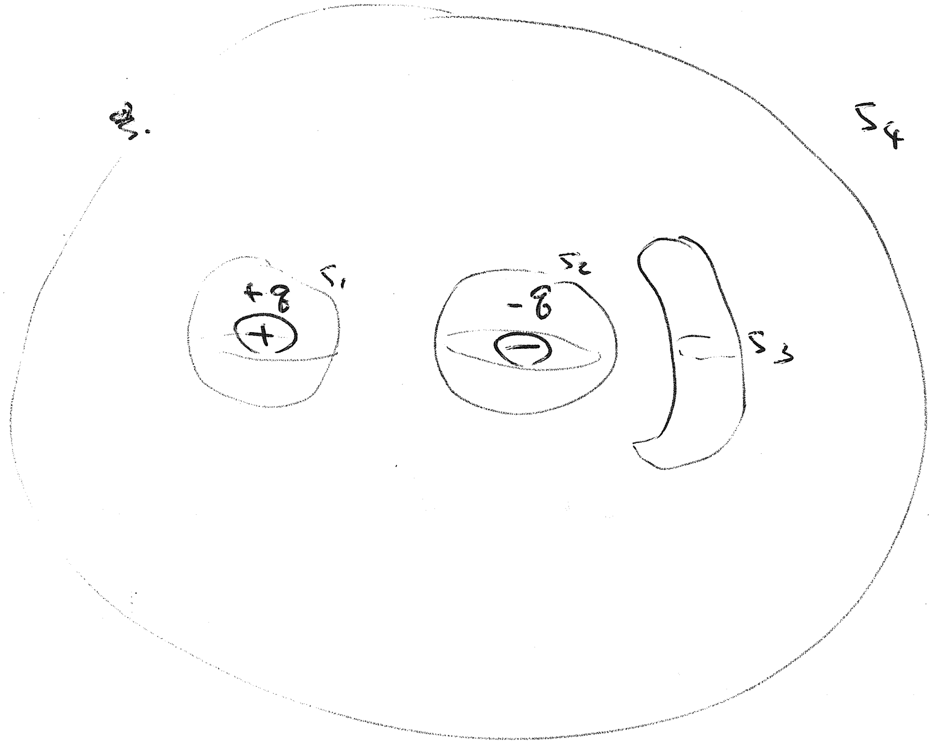






$$\Phi = \frac{q_1}{\varepsilon_0} + \frac{q_2}{\varepsilon_0} = \frac{q_1 + q_2}{\varepsilon_0} = \frac{q_{TOTAL}}{\varepsilon_0}$$

3.



$\Phi_{S_1} = \frac{q}{\varepsilon_0}$ $\quad \Phi_{S_2} = -\frac{q}{\varepsilon_0}$ $\quad \Phi_{S_3} = \frac{0}{\varepsilon_0}$

$\Phi_{S_4} = 0$

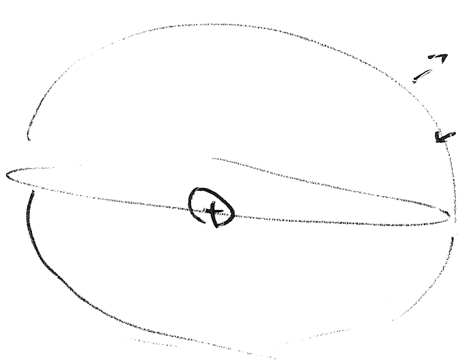
~~HOW DO WE USE IT? (TO CALCULATE $\vec{E}$?)~~

I.e. WHEN CAN WE USE IT TO CALCULATE $\vec{E}$?

1. IF YOU CAN DEFINE CONSTANT $|\vec{E}|$ SURFACE OVER WICH $\vec{E} \cdot d\vec{A}$ IS KNOWN
2. $\vec{E} = 0$ AT SOME SURFACE

1. CAN BE KNOWN ~~AS~~ BY SYMMETRY.

I.e.

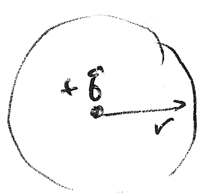


← SURFACE (SPHERICAL SYMMETRY)

EXAMPLE # 1

WHAT IS $\vec{E}$ AT THIS POINT?

1. DRAW SURFACE W/ CONSTANT $\vec{E}$



2. FIND ENCLOSED CHARGE

$$\underline{\underline{+q}}$$

3. APPLY GAUSS'S LAW

$$\text{AREA} = 4\pi r^2$$

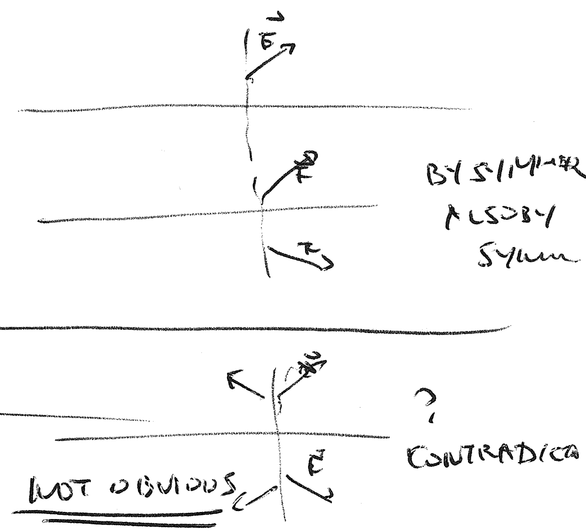
$$\Phi = E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\boxed{E = \frac{q}{4\pi \varepsilon_0 r^2}}$$

✓

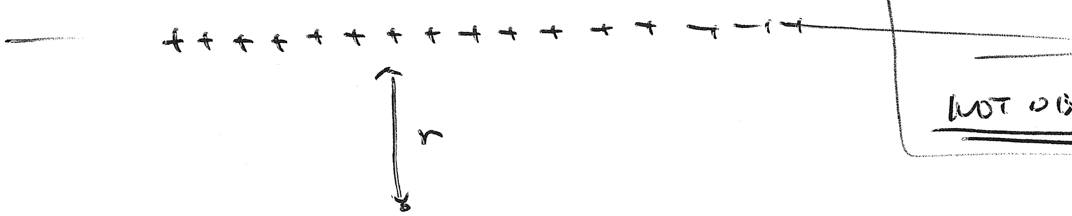
ASIDE

IF $\vec{E}$ NOT $\hat{r}$ THEN



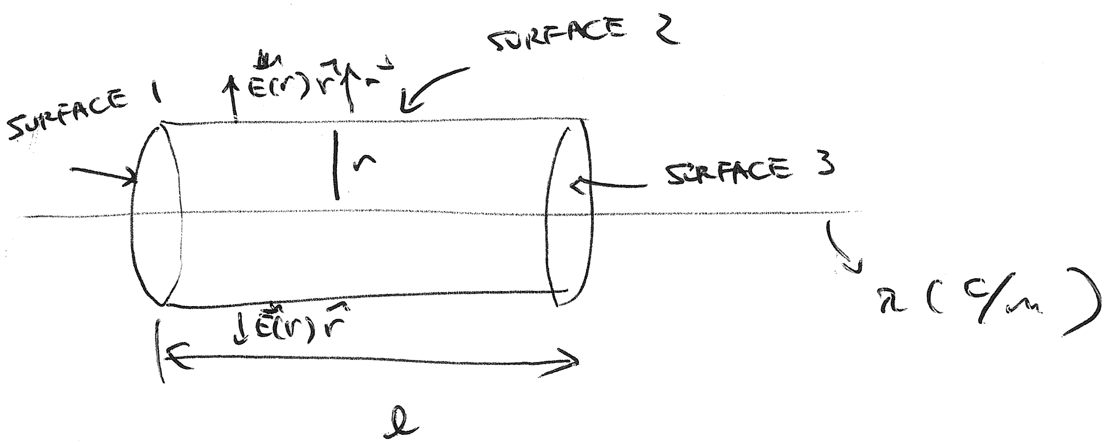
MORE COMPLEX EXAMPLE

INFINITE LINE CHARGE



CHARGE DISTRIBUTION CYLINDRICAL

SO IS THE $\vec{E}$.



SURFACE 1,3 : $\Phi_1 = \vec{E}_1 \cdot \vec{A}_1 = 0$

$\Phi_3 = \vec{E}_3 \cdot \vec{A}_3 = 0$

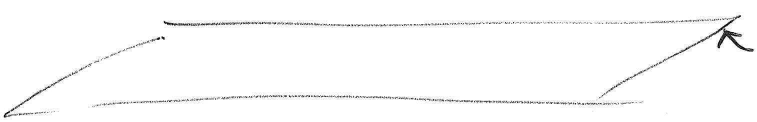
SURFACE 2 : $\Phi_2 = E \cdot A = E \cdot 2\pi r l = \frac{Q}{\varepsilon_0}$

$Q = \lambda \cdot l$

$$E\, 2\pi r \cancel{\ell} = \frac{\lambda \cancel{\ell}}{\varepsilon_0}$$

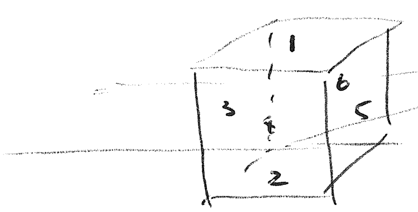
$$|\vec{E}| = \frac{\lambda}{2\pi\varepsilon_0 r} \Rightarrow \vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$$

EXAMPLE



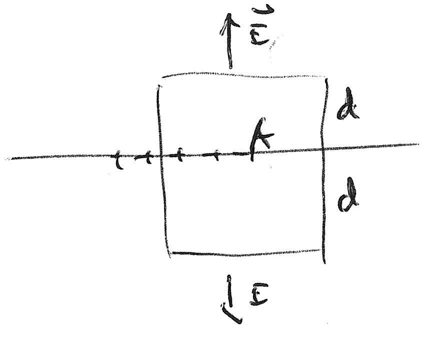
INFINITE CHARGE PLANE ~~INFINE~~

CHARGE AREA DENSITY = σ (C/m^2)



SURFACE TO USE

SURFACE 3-6 : $\Phi = 0$ AS $\vec{E}$ PERPENDICULAR TO SURFACE



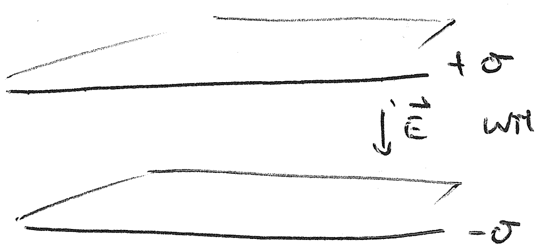
$$\Phi_1 = E(d) \cdot A$$

$$\Phi_2 = E(d) \cdot A$$

$$\Phi = 2E(d) \cdot A = \frac{Q_{\text{ENCLOSED}}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

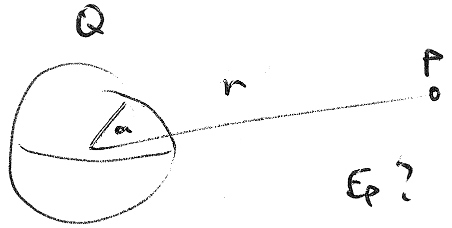
$$E(d) = \frac{\sigma}{2\varepsilon_0}$$

IN CLASS PRACTICE



WHAT'S $|\vec{E}|$?

SPHERICALLY SYMMETRIC CHARGE DISTRIBUTION

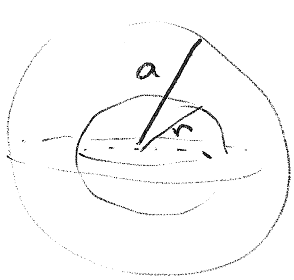


$r > a \Rightarrow \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$

$r < a$?

CHARGE DENSITY $= \rho$ (C/m^3)

$$\rho\frac{4\pi a^3}{3} = Q$$



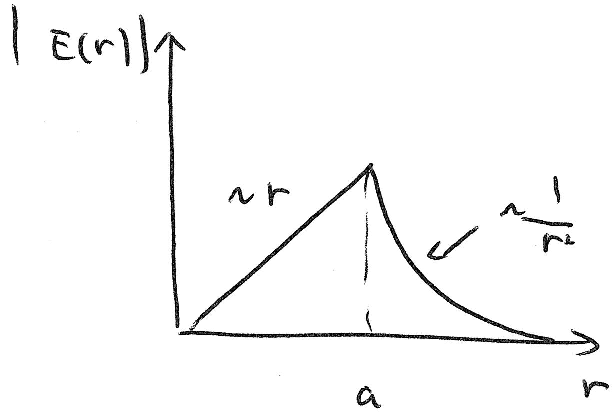
$$Q_{ENCLOSED} = \rho\frac{4\pi r^3}{3}$$

$$EA = \oint E\,\cancel{4\pi} r^2 = \frac{Q_{ENC}}{\varepsilon_0} = \frac{\cancel{4\pi} r^3}{3\varepsilon_0}\rho$$

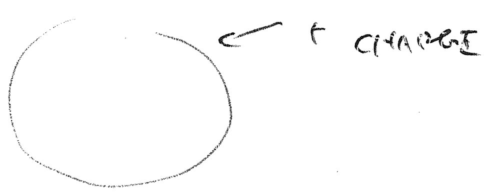
$$E = \frac{\rho r}{3\varepsilon_0}$$

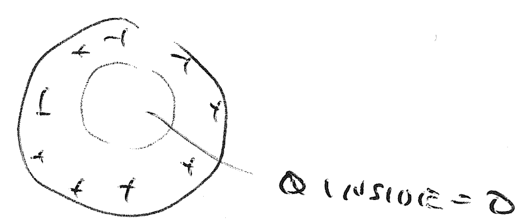
$r = a \quad E = \frac{\rho a}{3\varepsilon_0}$, or $\frac{1}{4\pi\varepsilon_0}\frac{Q}{a^2} = \frac{1}{\cancel{4\pi}\varepsilon_0}\frac{\cancel{4\pi} a^3 \rho}{3\cancel{a^2}}$

$$= \frac{\rho a}{3\varepsilon_0}$$ ✓



CONDUCTER



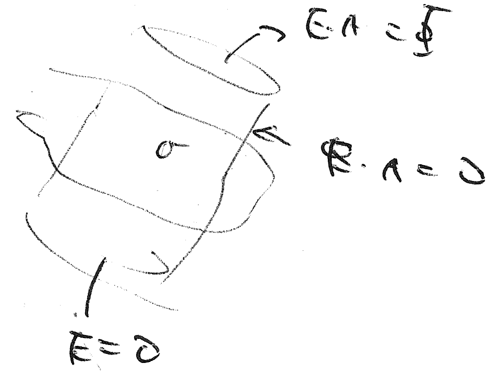


1. CHARGE IS AT SURFACE

2. $\vec{E}_{INSIDE} = 0$

3. RIGHT OUTSIDE OF CONDUCTOR, ELECTRIC FIELD IS σ/ε_0 → WE'LL PROVE NOW

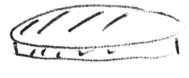
4. CHARGE DENSITY IS GREATEST WHERE THE RADIUS OF CURVATURE IS GREATEST → WE'LL PROVE LATR (NEXT WEEK)

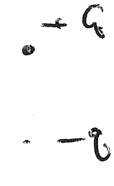


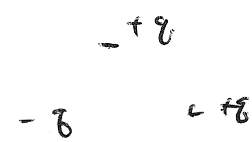
$$E \cdot A = \frac{\sigma A}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

YOU CAN'T USE GAUSS'S LAW TO CALCULATE ELECTRIC FIELD FOR







INSUFFICIENT SYMMETRY.